Longitudinal Shower Profile Reconstruction from Fluorescence and Cherenkov light

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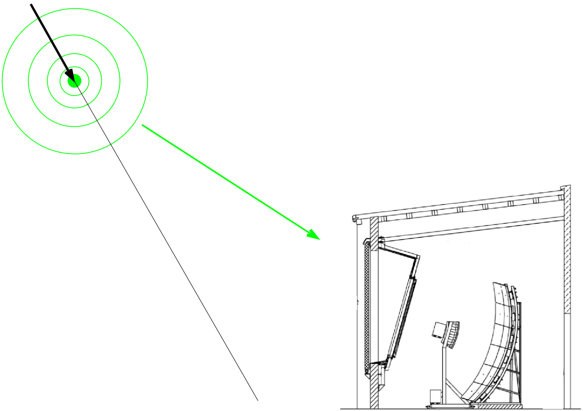
Light Production in UHECR air showers

- isotropic fluorescence emission
- forward beamed direct Cherenkov light
- Rayleigh- and Mie-scattered Cherenkov light



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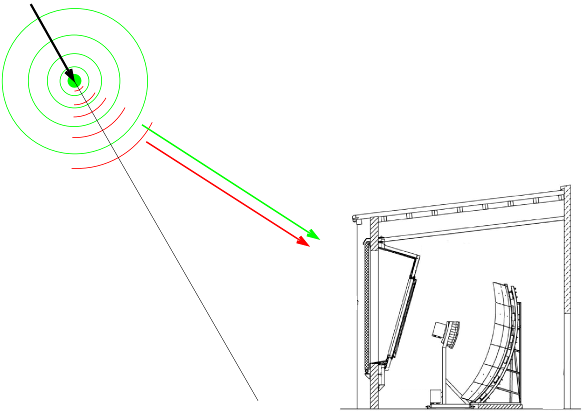

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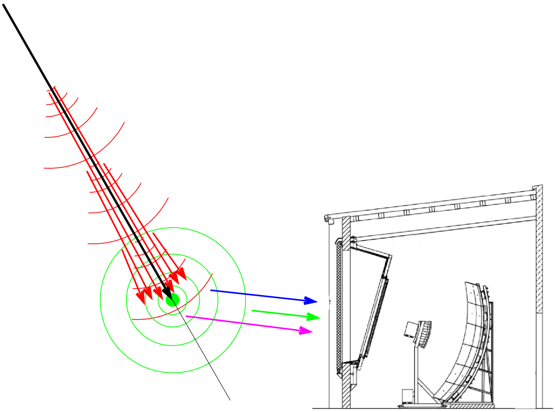

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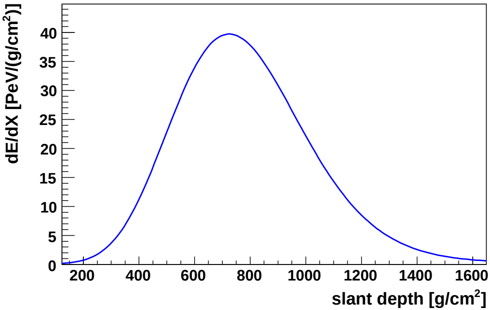
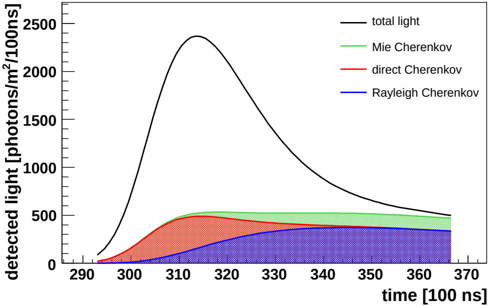


longitudinal profile

↓



light at detector

simulation: FDSim (L. Prado et al., NIM A545 (2005), 632)

Light Production in UHECR air showers

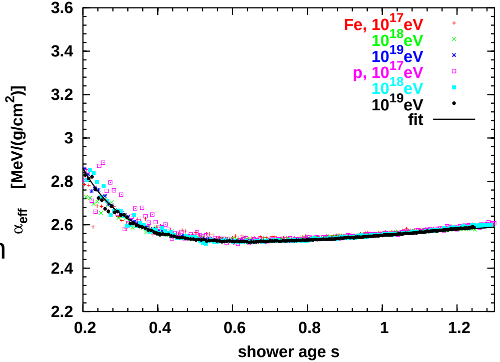
Note:

- Fluorescence yield $\propto$ dE/dX
- Cherenkov yield $\propto N_e$

but:

unversality of secondary electron spectra
$\rightarrow$ average energy deposit per electron universal!

$$\rightarrow dE/dX = \alpha_{\text{eff}}(s) \cdot N_e$$



F. Nerling et al., Astropart. Phys. 24 (2006), 421
(see also: M. Giller et al., J. Phys. G 30 (2004), 97)

Traditional ansatz
(R. M. Baltrusaitis et al., NIM A240 (1985), 410)

Iterative procedure:

step 1: assume no Cherenkov contribution and calculate N_e
step 2: calculate Cherenkov contribution
step 3: subtract Cherenkov contribution from light at diaphr.
step 4: recalculate N_e and go to step 2

Problems:

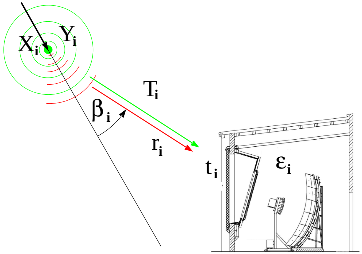
- iteration stops if $N_e < 0$ after step 3
- slow convergence
- calculation of statistical uncertainties?

Fluorescence and direct Cherenkov light

direct relation between light flux n_i at time t_i and energy deposit $dE/dX_{\,i}$ at depth X_i

$$n_i^d = \underbrace{\frac{T_i\,\varepsilon_i}{4\pi r_i^2}}_{\text{attenuation to detector}} \cdot \underbrace{\left[Y_i^f + Y_i^C(\beta_i)/\alpha_i\right]}_{\text{light yields}} (\mathrm{d}E/\mathrm{d}X)_i$$

$$\equiv d_i \cdot (\mathrm{d}E/\mathrm{d}X)_i$$



Y_i: Cherenkov and fluorescence yield
α_i: average dE/dX per electron
T_i: light attenuation to detector
ε_i: detector efficiency

(wave length dependence not shown for clarity)

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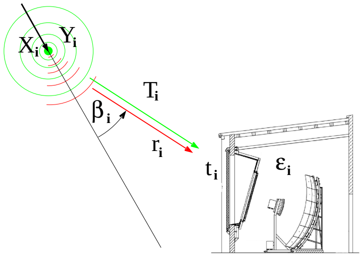

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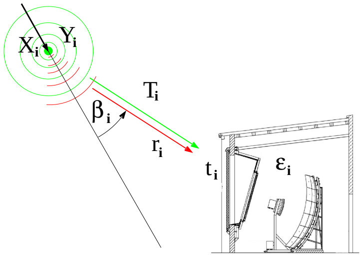

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$$\begin{pmatrix} n_1 \\ \vdots \\ n_i \\ \vdots \\ n_n \end{pmatrix}_d = \begin{pmatrix} d_1 & 0 & \dots & & 0 \\ 0 & \ddots & & & 0 \\ \vdots & & d_i & & \vdots \\ 0 & & & \ddots & 0 \\ 0 & 0 & \dots & 0 & d_n \end{pmatrix} \begin{pmatrix} (\mathrm{d}E/\mathrm{d}X)_1 \\ \vdots \\ (\mathrm{d}E/\mathrm{d}X)_i \\ \vdots \\ (\mathrm{d}E/\mathrm{d}X)_n \end{pmatrix}$$

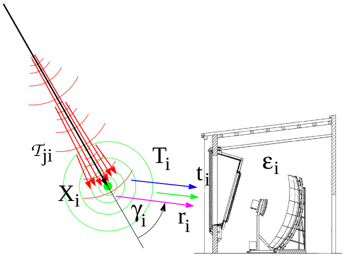
$\rightarrow$ simple diagonal matrix equation

Scattered Cherenkov light

scattered Cherenkov light at detector:

$$n_i^s = \underbrace{\frac{T_i\,\varepsilon_i}{4\pi\,r_i^2}}_{\text{attenuation to detector}} \cdot \overbrace{f(\gamma_i)}^{\text{scattering to detector}} \cdot \underbrace{\sum_k Y_k^C/\alpha_k\,\mathcal{T}_{ki}\,(\mathrm{d}E/\mathrm{d}X)_k}_{\text{Cherenkov beam}}$$

$$\equiv \sum_k s_{ik} \cdot (\mathrm{d}E/\mathrm{d}X)_k$$



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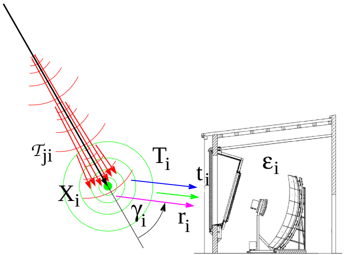

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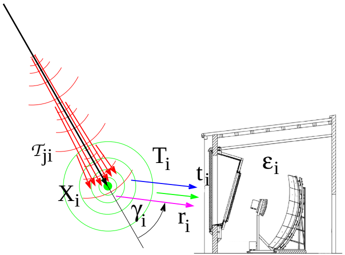

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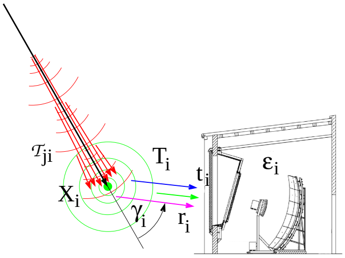

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Scattered Cherenkov light

scattered Cherenkov light at detector:

$$\begin{pmatrix} n_1 \\ \vdots \\ n_i \\ \vdots \\ n_n \end{pmatrix}_s = \begin{pmatrix} s_{11} & 0 & \dots & 0 & 0 \\ s_{12} & \ddots & & & 0 \\ \vdots & & s_{ii} & & \vdots \\ s_{1(n-1)} & & & \ddots & 0 \\ s_{1n} & s_{2n} & \dots & s_{(n-1)n} & s_{nn} \end{pmatrix} \begin{pmatrix} (\mathrm{d}E/\mathrm{d}X)_1 \\ \vdots \\ (\mathrm{d}E/\mathrm{d}X)_i \\ \vdots \\ (\mathrm{d}E/\mathrm{d}X)_n \end{pmatrix}$$

$\rightarrow$ triangular matrix equation

Total light at detector

sum of direct and scattered light:

$$\begin{pmatrix} n_1 \\ \vdots \\ n_i \\ \vdots \\ n_n \end{pmatrix}_{\text{tot}} = \begin{pmatrix} s_{11}+d_1 & 0 & \dots & 0 & 0 \\ s_{12} & \ddots & & & 0 \\ \vdots & & s_{ii}+d_i & & \vdots \\ s_{1(n-1)} & & & \ddots & 0 \\ s_{1n} & s_{2n} & \dots & s_{(n-1)n} & s_{nn}+d_n \end{pmatrix} \begin{pmatrix} (\mathrm{d}E/\mathrm{d}X)_1 \\ \vdots \\ (\mathrm{d}E/\mathrm{d}X)_i \\ \vdots \\ (\mathrm{d}E/\mathrm{d}X)_n \end{pmatrix}$$

in short:

$$\boxed{\mathbf{n} = \mathbf{C}\,\mathbf{w}}$$

$\rightarrow$ triangular matrix equation

Solution of Cherenkov-Fluorescence equation

Cherenkov-Fluorescence equation:

$$\mathbf{n} = \mathbf{C}\,\mathbf{w}$$

is solved by matrix inversion

$$\mathbf{w} = \mathbf{C}^{-1}\mathbf{n}$$

Solution has covariance matrix

$$\mathbf{V_e} = \mathbf{C}^{-1}\,\mathbf{V_n}\left(\mathbf{C}^{\mathrm{T}}\right)^{-1}$$

$\rightarrow$ correlation due to Cherenkov beam

Conclusions

new algorithm for longitudinal profile reconstruction:

- Cherenkov light is signal, not background
- light at detector depends *linearily* on shower size
- linear least square solution
- robust, unbiased and fast

additional effects to be considered:

- wave-length dependence of fluorescence and Ch.-light
  $\rightarrow \sum \mathbf{n}_\lambda = \sum \mathbf{C}_\lambda \mathbf{w}_\lambda$
- extrapolation outside field of view
  $\rightarrow$ four parameter Gaisser-Hillas fit
- age-dependence of Cherenkov light production
  $\rightarrow$ one iteration needed

see proceedings for further details!